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How Willingness to Pay leads to Public Choice

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1 Introduction

On a free market individuals exchange their goods to increase their utility. The market mechanism (under perfect competition and information) normally leads to a Pareto efficient allocation for private goods: Individuals exchange goods and services until no ones' utility can be increased without diminishing the utility of somebody else (Arrow and Debreu 1954; Mas-Colell, Whinston, and Green 1995). However, for public goods, this mechanism fails. Although individuals have a preference for the provision of public goods, their "genuine" willingness to pay for the good will not be revealed on the market due to the free-rider problem (Samuelson 1954; Olson 1971; Brubaker 1975; Sandler 1992). Only a small share of this willingness to pay – to fulfill social incentives such as silencing one's conscience or gaining reputation (Gogoll and Schlieszus 2021) – will be revealed on the market. The free-rider problem results in a provision of public goods via the market mechanism, which is below optimum. However, the existence of public goods, such as national defense, infrastructure or even climate is crucial for the individual utility. The question remains, if and (if so) how individuals can satisfy their preferences for public goods if not via the regular market mechanism.

For small public goods (pubic goods consumed by a limited number of individuals in a small area) free-riding will be less dominant (if it exists at all) due to reciprocity (Axelrod 1984; Nowak and Sigmund 2005; Ule et al. 2009; Mani, Rahwan, and Pentland 2013). Social incentives such as the fear of social sanctions or the hope of social benefits (Olson 1971; Kreps and Wilson 1982; Ostrom 1990; Bornstein, Erev, and Rosen 1990), but also social incentives not linked to reactions of other individuals e.g. altruism (Margolis 1983; Taylor 1987; Guagnano, Dietz, and Stern 1994) can stabilize the revealed willingness to pay. Especially for large public goods that many people can access across a country, region or the world this mechanism fails. Instead, an entity is needed which forces individuals to pay and that uses the collected money to provide public goods. This might be a government, which collects taxes and determines the level of public goods provided. However, this level is neither set arbitrarily nor independently of individuals' choices since in democracies the government is elected by the same individuals.

Public choice literature i.e the median voter theorem explains this process of provision of public goods (Black 1948; Downs 1957; Black 1958; Holcombe 1989; Batina and Ihori 2005). The theorem uses the voters' preferences and budgets to establish a convergence process of parties, which leads to the provision of public goods. However, it is not possible to model the supply within this framework. I tried to solve this problem by implementing the willingness to pay in a common price-quantity diagram. This representation helps with modeling the supply and makes it easier to analyze changes of preferences and budgets.

In this paper, I aim to show with a simple microeconomic model, how the willingness to pay determines a voter's choice of a political party. In contrast to a "common" household optimization, for individuals it is not possible to simply pick the optimal level of a public good. The level of the public good is the same for all households within a state. Thus, I will analyze how individuals maximize their utility choosing between non-optimal combinations. This optimization behavior is reflected in the voting decision of each individual. Using this calculus and including a supply side that produces the public good enables to analyze the behavior of politicians as self-interested mediators between supply and demand. Finally, I aim to answer the question: How does the individuals' willingness to pay lead to the provision of public goods?

For this purpose, in chapter two I develop a simple microeconomic model in several steps: First, I analyze how a voter will optimally allocate her budget given a government and a certain level of the public good. Second, I will analyze which party a voter will choose, given arbitrarily proposed combinations of quantity and price of the public good (which determines the amount of private goods an individual can consume). Third, I will explain how this calculus of the voter determines the provision of the public good in the cases of a political monopolist, perfect competition and imperfect competition on the political market. Finally, some possible implications of the model are drawn.

2 Model

2.1 Utility Maximization for a Fixed Level of a Public Good

At first, I want to show how an individual maximizes its utility given a fixed level of a public good. This can be shown by starting from the perspective of one utility-maximizing individual. Let the utility be given by a common Cobb-Douglas function

$$U(x_c, x_p) = x_p^{\alpha} x_c^{\beta} \tag{1}$$

with one private good x_p and one public good x_c. α shows the preference for the private good relative to the preference for the public good β. For simplicity, the private and the public good will not be differentiated further. Assuming that

$$0 < \alpha < 1 \text{ and } 0 < \beta < 1 \tag{AS1}$$

ensures that both goods are part of the utility function. Using monotonic transformation allows to assume that

$$\alpha + \beta = 1. \tag{AS2}$$

The budget restriction is given by

$$m = x_p p_p + p_c x_c. \tag{2}$$

Public goods are characterized by the properties of non-excludability and non-rivalry in consumption. Every unit of the public good x_c is available to all individuals. Thus, the price for one unit of the public good does not have to be paid in total by each individual. Let the price for one unit of the public good p_c be the individual share of the total price for one unit of the public good. This share is given by the price for one unit divided by the number of tax payers.

Compared to a "common" household optimization with respect to two private goods, the individual cannot choose the level of the public good it wants to consume. The consumption of the public good is equal for each individual within a state. Let the level of the public good be set by the government. Every citizen has to pay the poll tax $p_c x_c$. The individual's optimal response is to take the remaining part of the budget, which is not needed for financing x_c, and using it for the consumption of x_p.

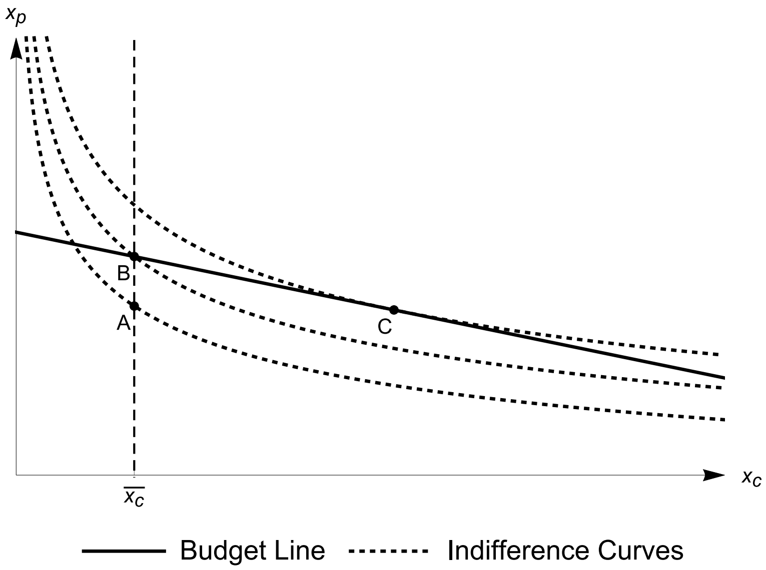

Figure 1: Household optimization with quantitative restriction

Assume that the provided level of the public good is given by $\overline{x_c}$. Now consider for exemplary points A, B and C. Starting from point A, the individual can achieve a higher utility (indifference curve) if it spends more on the private good x_p. At point B, the budget is completely spent. The only option to further increase the utility (without exceeding individual's budget) is by shifting consumption from the private to the public good, e.g., point C. However, this is not possible due to the fixed consumption level of the public good. Thus, the best response for a given level of $\overline{x_c}$ is given by

$$x_p^* = \frac{m - \overline{x_c} p_c}{p_p}. \tag{3}$$

2.2 Selection of the Utility Maximizing Combination

Different political parties propose different (for now randomly selected) combinations of a level/amount x_c and price p_c of the public good. $x_c * p_c$ is the total amount each individual has to pay for the proposed level of the public good. This amount is collected as income independent poll tax. Knowing how an individual will maximize its utility for a given level of the public good, let it now choose between these different proposed combinations. The individual elects the party that proposes a combination that is maximizing its utility.[1] The elected party collects the announced poll tax and provides the proposed combination. Thereby, the product $x_c * p_c$ times the number of tax payers represents the maximum amount which can be used for financing this level of the public good.

Which combination will be chosen by this individual, if it can choose between different proposed combinations of parties? To answer this question, let us compare five proposed example combinations D to H (see figure 3). At first, we can consider the individual's budget. The curve that determines the maximal affordable consumption level of x_c is given by dividing the budget m by the price of the public good p_c. Solving for x_c yields

$$p_c = \frac{m}{x_c}.$$

This provides the budget curve (see figure 2).

[1] For simplicity, strategic voting behavior will not be considered.

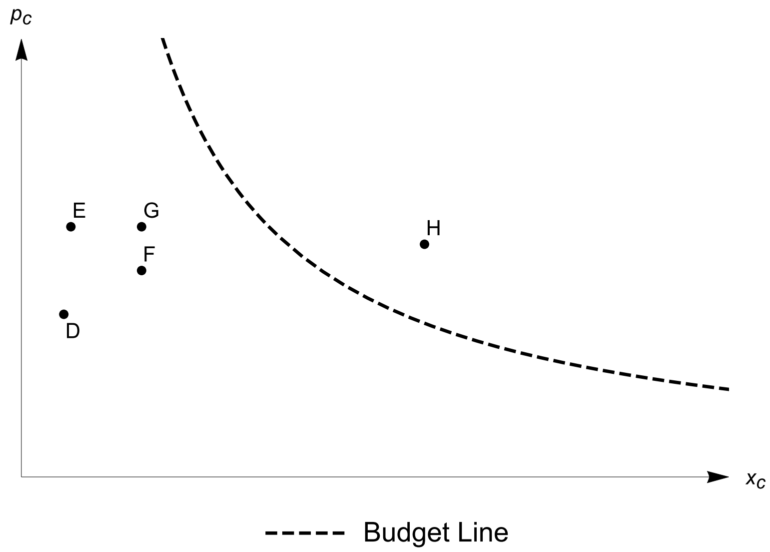

Figure 2: Budget curve

On this curve the budget is completely spent for the public good. The resulting utility is zero as no private goods can be consumed. Combinations above the budget curve are not possible in this model. The poll tax would exceed the budget of the individual. Thus, combination H in figure 2 can be ruled out.

A second way of comparing the remaining combinations is by implementing a common demand curve (see figure 3)

$$D(p_c) = \frac{\beta m}{(\alpha + \beta) p_c}. \tag{4}$$

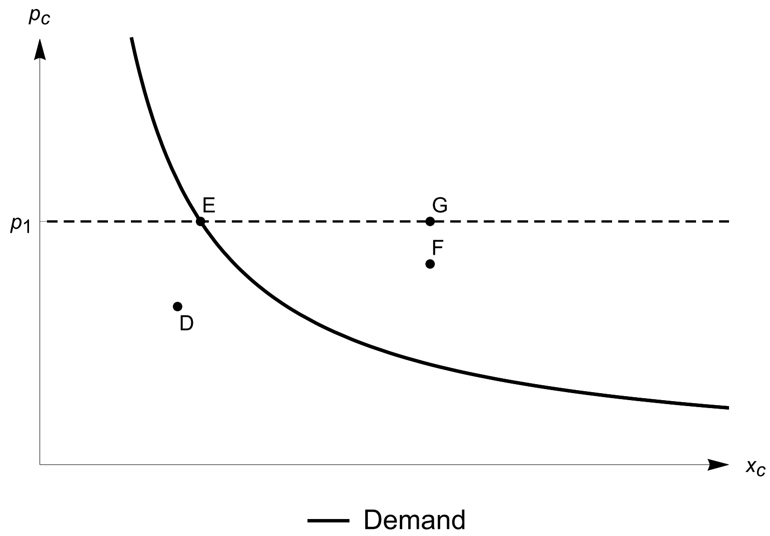

Figure 3: Demand curve

For regular household optimization, this curve shows for a given price p_c the utility-maximizing level of the considered good. However, that does not imply that the utility for the individual is equal everywhere on the demand curve. On the contrary: The utility along the demand curve increases as the price decreases (proof see Annex A.1). An individual would prefer any point on the demand curve with a lower price. Thus, if multiple combinations are proposed on the demand curve, those with higher prices can be ruled out. For a fixed level of the public good, an individual would always prefer the combination with a lower price (proof see Annex A.2). Now consider two combinations with the same (unit) price, but different levels. Assume that the levels are both lower or both higher than the level on the demand curve. The combination closer to the demand curve gives a higher utility as preferences are single-peaked (proof see Annex A.3). Thus, we can rule out point G as a possible choice of the considered voter (see figure 3).

It is likely that not all proposed combinations are lying on the demand curve or have the same price or the same level of the public good as another combination. Therefore, the only reasonable measure for evaluating the remaining combinations is the direct comparison of the resulting utilities. Calculating the respective utility yields that combination D gives the highest utility ($U_D = 65; U_E = 62; U_F = 59$)[2] and is thus chosen by the voter. For a more general and non-discrete comparison of different combinations, all points with

[2]Values calculated by setting $\alpha = 0.8$, $p_p = 7.5$ and $m = 1000$.

the same utility can be depicted on an "indifference curve" in the price-quantity diagram. Using the utility function (equation 1) and substituting the private good x_p by the optimal response function for the respective good (equation 3), yields a utility function that only depends on the public good x_c,

$$U(x_c) = \left(\frac{m - x_c p_c}{p_p}\right)^{\alpha} x_c^{\beta}. \tag{5}$$

Setting this expression equal to a constant utility value ($\overline{U}$) and solving for the price p_c yields the respective indifference curve[3] for a constant utility value:

$$p_c = \frac{m - p_p(\frac{\overline{U}}{x_c^{\beta}})^{\frac{1}{\alpha}}}{x_c} = I_{\overline{U}}. \tag{6}$$

Using the calculated utility of the three combinations and implementing the respective indifference curves gives figure 4:

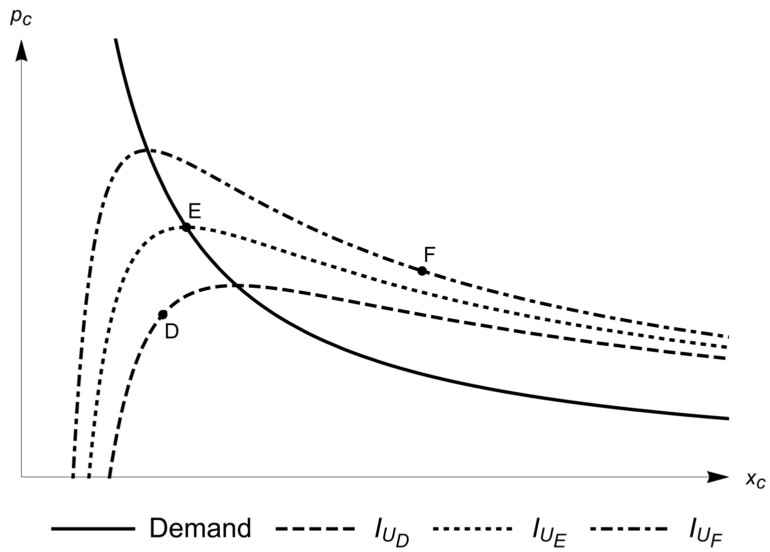

Figure 4: Indifference curves in the price-quantity diagram

A comparison of the three different indifference curves shows that the higher the indifference curve, the lower its utility. This is in line with the previous observation that combinations on the demand curve have a higher utility the lower the price. Each

[3]It is important to mention that in contrast to a "common" indifference curve this one includes the budget, which is needed to calculate the consumption of the private good.

indifference curve has its maximum price on the demand curve. For a given utility, this point represents the highest price possible. If the consumer has to consume an amount of the public good which is not on the demand curve, the price of the public good must be lower to keep utility constant. The lower price compensates for the undesired level of the public good that the individual has to consume. This compensation results from the possibility to buy more public goods (points right to the optimum) or more private goods (left to the optimum). The deducted indifference curves in the price-quantity diagram enable us to compare every proposed combination of political parties. Now, it is possible to investigate whether and how parties anticipate voters' optimization to maximize their votes.

2.3 Supply of Parties

So far, the combinations of the public good, proposed by different parties, were just arbitrarily chosen. However, it is reasonable that the selection of these combinations by parties follows a certain calculus. Public choice literature suggests that politicians want to (mis)use their power to maximize their own welfare (Downs 1957).[4] This can be done by collecting more taxes for the provision of public goods than actually needed. The difference between the cost of production for the public good and collected tax revenues can be used for increasing their welfare.

For calculating this difference we have to implement the supply of public goods into the model. Let us assume that we have a perfect competitive market where different firms produce the public good. For simplicity, we further assume that the public good can be produced at constant marginal cost. Fixed cost are included in the marginal cost (long-term perspective) or absent. As we consider public goods, the individual marginal cost for a unit c_c is equal to the total cost for a unit divided by the number of taxpayers.

[4]With his book "An economic theory of democracy" Downs (1957) was the pioneer of analyzing political behavior based on the utility maximization of politicians. Many further literature relates to and is based on his ideas. Another important model in this context is the budget-maximizing model developed by Niskanen 1971. In comparison to Downs, he focuses on budget-seeking of bureaucrats, which are not investigated in this model.

To increase their welfare, politicians can propose a level of x_c at a higher price than production cost (see combination G in figure 5). The difference μ, which is the price they get from the taxpayer p_c minus the price that they have to pay for the production of the public good c_c, aggregated over all provided public goods, represents their welfare gain (striped square). For an individual, it represents a loss of utility as the individual can spend less on private goods. This difference can be considered as an inefficiency.

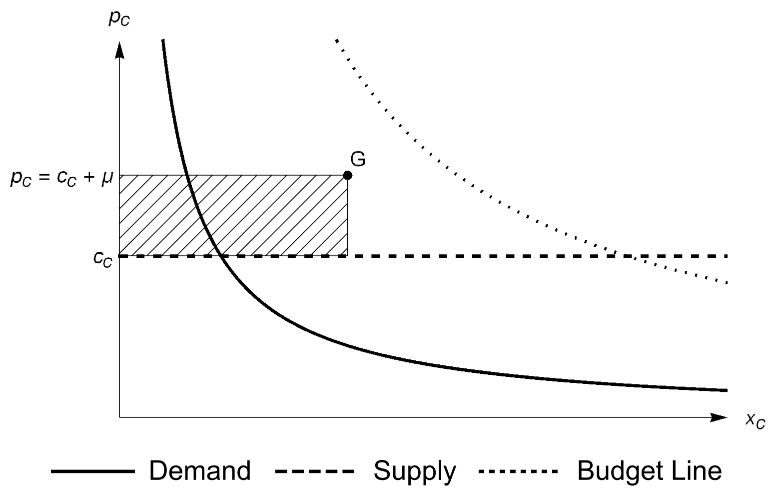

Figure 5: Rent-seeking of politicians

The following subchapters aim to analyze how different types of markets (monopolist, perfect competition, imperfect competition) lead to the provision of public goods. Is and (if so) in which way is the inefficiency maximizing calculus of politicians restricted and guided by the willingness to pay of individuals?

2.3.1 Monopolist

Let us consider the situation of a monopolist on the political market. Having a political monopolist implies that there are no other parties and there is no competition and no reelection restriction (autocracy).

Compared to a market of private goods with one monopolist, people do not have the freedom to stop their consumption. They are forced by law to pay the tax, which is set by the government. However, they can refuse to pay it. This is in line with utility-maximization calculus if the sanction multiplied with the probability of detection is lower than the price p_c multiplied with the amount x_c set by the monopolist (Becker 1968). Furthermore, they can leave the country (exit option) to escape high tax rates. Compared to the avoidance cost a private monopolist induces, these options are costly for individuals. As a consequence, limiting the power of the government by legal restrictions seems to be important for a society. Bad governance resulting in a dysfunctional political competition is decreasing the welfare of a society massively.

Let us determine the optimal provided level of the public good for a political monopolist. Looking at only one individual, the political monopolist can enlarge its rent by increasing the price p_c. In order to fully obtain the individual's budget it is reasonable to focus only on combinations on the budget curve. Revenues are equal for every combination on the budget curve. The highest profits can be generated by providing only slightly more than zero public goods as the aggregated costs are at the lowest due to the small amount of produced public goods. However, for the people's utility, it is the worst case. It is likely that more and more people will try to choose the exit option, avoid paying taxes, or have an incentive to work against the political monopolist by building an opposition and/or starting a revolution. In this model, the individual marginal cost for one unit of the public good c_c would increase as the number of tax payers decreases. This would lead to a reduction of inefficiencies of politicians as more of the revenues are needed to finance the public good.

For sure, the political monopolist has an incentive to take measures to prevent such behavior. However, this induces costs in addition to the costs caused by successful tax evasion. Therefore, it seems to be inefficient to increase the price too heavily for only a small level of the public goods. Enlarging the provided amount of public goods in combination with a small mark-up for every unit, might be a more efficient strategy. If the monopolist provides a unit cheaper (i.e. where marginal costs are lower than the imposed tax rate), his accumulated rent increases with every additional supplied unit. Thus, it is likely that in non-competitive political markets, the government will provide an excessive amount of public goods, especially those where they can secretly set a price that is drastically above the production cost.

2.3.2 Perfect Competition

Let us switch from a political monopolist to a perfect competitive political market. In contrast to a political monopolist, parties have to win an election first. For simplicity, we use a first-past-the-post electoral system, where the party with most votes wins an election and forms the government. Thus, it is always the primary goal of parties to garner the most votes among all parties.

Parties can propose a combination on the supply-curve or above. Combinations below the curve are not affordable as the tax revenues are not high enough to cover the cost of production. However, parties could try to attract voters by proposing such combinations. If parties only want to be elected one time ("one shot game"), voters might believe that the proposed combination is offered. In more stage games with a competitive political market, this strategy is likely to fail as individuals will not reelect a party that did not keep their promises and elect a different one. For simplicity, we assume perfect information s.t. the proposed combinations are also finally offered.

If a party proposes a combination above marginal cost of production, another party has an incentive to provide the same amount at slightly lower prices. The voter will always vote for the party that proposes the same level at a lower price since utility is higher for the individual (see previous section). This competition will result in an efficient provision of public goods at cost of production. The result is identical to Down's efficiency theorem (Downs 1957).

The question remains which level of the public good parties will propose. If there is just one voter (or all voters are equal regarding their preferences and budget), a party will propose its combination at the intersection of the supply and demand curve. Every other combination has a lower utility for the consumer or is not feasible.

Let us now take a look at a situation with three voters having the same preferences but different budgets. The resulting three demand curves can be seen in figure 6.

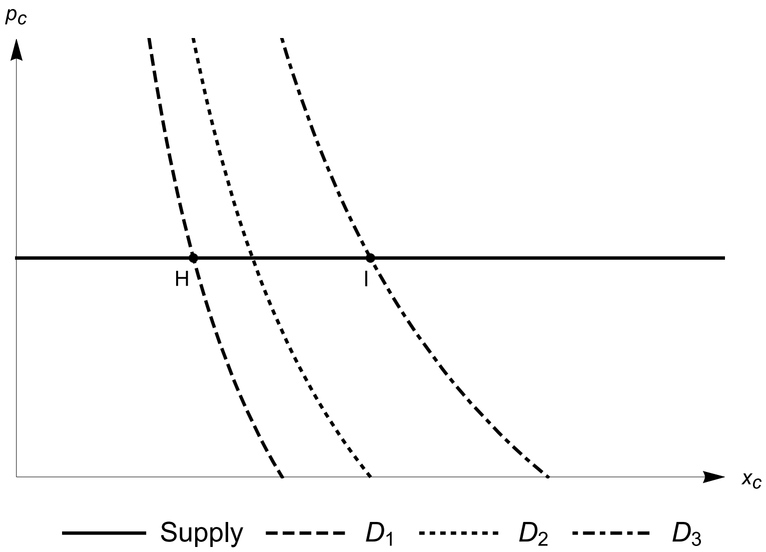

Figure 6: Different individuals and proposed combinations of parties

Individual three (one) has the highest (lowest) budget resulting in the highest (lowest) willingness to pay for the public good. Assume that we have two parties, each proposing a combination of public goods at marginal cost of production. Let party one start from point H, party two from I. In this situation, voter one would vote for party one and voter three for party two as it is their optimal choice. For voter two, both combinations are not optimal. We can investigate its voting choice by comparing its indifference curves for the two given combinations (figure 7).

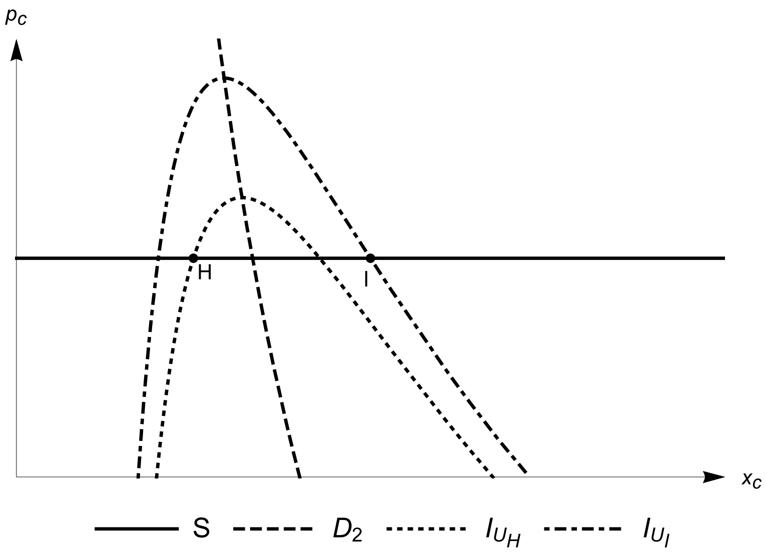

Figure 7: Indifference curves of voter two

The dotted and dot-dashed curve show the two indifferent curves with the respective utility that voter two would get if party one (combination H) or two (combination I) is elected. The utility of voter two is higher if party one wins the election and combination H is provided (lower indifference curve). Party two anticipates the voter's choice and has an incentive to change the proposed combination to win the election. Voter two will switch his choice if the combination leads to a higher utility. This is the case if party two proposes a combination between the intersections of the indifference curve of combination H with the marginal cost function. For example, this is combination K in figure 8.

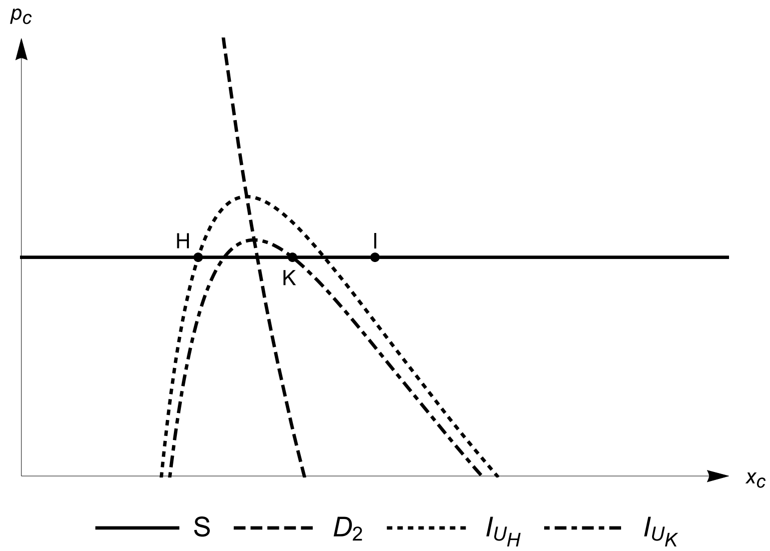

Figure 8: Anticipation of losing party

If party two provides combination K, voter two will switch, and party two will win the election if the choice of voter one and three remains unchanged. This condition holds as the utility of voter one and three are monotonously decreasing for all combinations beside their maxima at the intersection of their demand and supply (see proof Annex A.3). Due to the change of the proposed combination of party two, party one would now loose the election and has an incentive to change its proposed combination as well. Crucial is again voter two. If party one would, for example, try to get the vote of voter three, she will always lose voter one and not win the election if she does not convince voter two as well. Thus, party one will choose a combination between the intersections of the indifference curve of combination K with the marginal cost function. This process of anticipation leads to a convergence to combination L, the optimal combination of voter two, see figure 9.

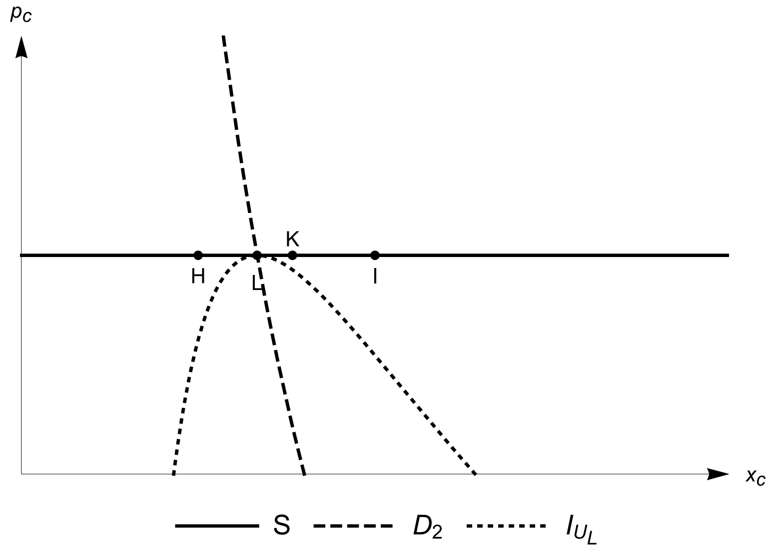

Figure 9: Convergence to median voter

If both parties propose combination L, they are in equilibrium, as they do not have an incentive to change their combination. Any other combination would lead to losing the vote of voter two and result in losing the election. This convergence process leads to the same result as the median-voter theorem, i.e., parties will always propose the combination of the median voter (see Hotelling 1929; Black 1948; Downs 1957; for a literature review of median voter theorem see Holcombe (1989), Batina and Ihori (2005), and Adams, Merrill, and Zur (2020)).

So far we have used the strong assumption of constant marginal cost of production for the public good. However, it is possible that marginal production cost are increasing with the level of the public good. For example, for an environmental public good like climate, for a higher level of the public good emissions have to be decreased. Whereas the first unit of emissions can be avoided at quite low cost, for every further unit the marginal abatement cost increases. Thus, figure 10 shows a supply function with increasing marginal cost.

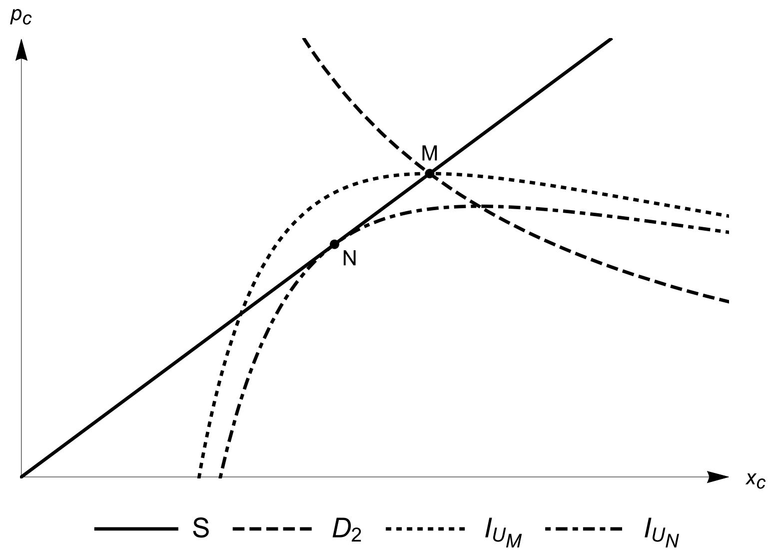

Figure 10: Convergence to median voter at increasing marginal cost

For constant marginal cost of production the intersection of the supply and demand of the median voter determines the utility maximizing combination of the voter. This equilibrium gives a utility for individual two that is demonstrated by the indifference curve I_{U_M}. In contrast to the intuition, this is not the utility-maximizing point for voter two on the marginal cost curve. He would prefer a lower consumption of the public good x_c. The utility-maximizing point for individual two is reached at point N.

The process of convergence of parties to the combination of the median voter is constant though. Each individual has its maximum on the marginal cost curve. Beside its utility maximum the utility is monotonously decreasing on the marginal cost curve, hence the path that parties will propose combinations on (proof see Annex A.4). Thus, the same process of anticipation of parties and convergence results. Only the final proposed combination of the parties is at point N and not at point M, the intersection of the demand and supply curve.

2.3.3 Imperfect Competition

Following the literature, the median voter theorem holds only for two parties (Rowley 1984). A higher number of parties does not lead to a stable equilibrium (except with very strong assumptions) (Adams, Merrill, and Zur 2020). With only two parties proposing combinations, it seems reasonable to assume that they have some scope to generate inefficiencies. The start remains the status quo situation as before (3 voters, 2 parties 1 & 2 proposing combinations H & I respectively, see figure 7). In addition, let us assume that they produce as many inefficiencies as possible in order to increase their welfare. However, their first goal remains winning the election, so that they are able to collect taxes to misuse them. As before, party two wins the election as voters one and two vote for this party. In contrast to the previous situation, party two would not propose a combination on the marginal cost curve. To win the election and gain welfare by inefficiencies, it has to choose a combination within the area between the intersections of the indifference curve of combination H and the marginal cost curve (see striped area in figure 11).

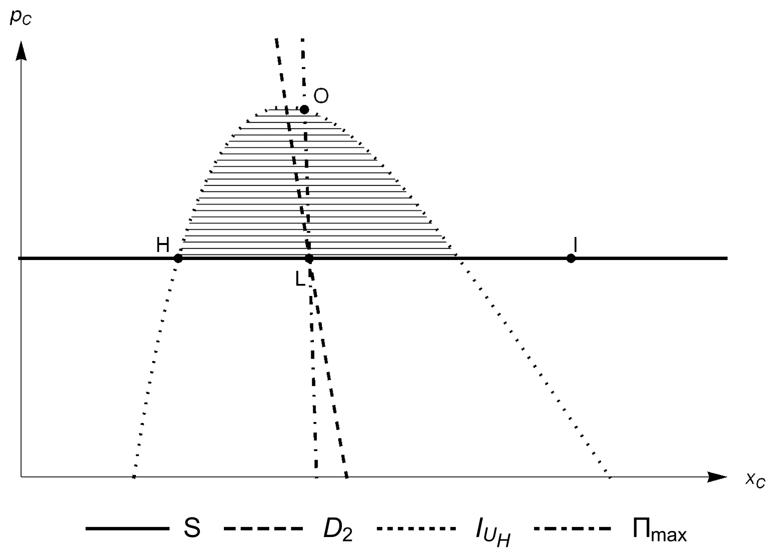

Figure 11: Convergence with inefficiencies

This combination will no longer be on the marginal cost curve but will have a higher price in order to maximize inefficiencies. The function

$$\Pi_{max}(x_c) = \frac{\beta m - \alpha c x_c}{\beta x_c} \tag{7}$$

shows the inefficiency-maximizing combination for each utility of voter two (for proof see Annex A.5). Using this formula the combination of maximal inefficiencies in this area at point O in figure 11 can be determined. If party two proposes this combination voter two is indifferent between electing party one and two as combinations at points H and O give her the same utility. Thus, a combination on the inefficiency maximizing curve right below combination O leads to winning the election. At the same time, this point is maximizing inefficiencies. To win the election while maximizing inefficiencies, party one would now calculate this inefficiency-maximizing point of the area within the intersections of the indifference curve of the new combination of party two and the marginal cost curve.

The utility of voter two (and all other voters; for proof see Annex A.6) increases for combinations along the inefficiency maximizing curve closer to the optimal bundle of the median voter. Thus, a convergence process along this function starts which finally leads to the provision of the combination of the median voter. This implies that even for two parties allowing them to maximize their inefficiencies under the "election restriction" an efficient provision without inefficiencies results.

3 Implications

With this willingness to pay for public goods it is possible to evaluate the influences on the provision of public goods by political parties. As shown above, the parties will provide the combination that is at the utility-maximizing point of the median voter. This enables us to analyze the drivers of the provision of public goods.

Let us first consider changes of the median voter while still being the median voter. With a shift of preference or income the considered individual will remain in its relative position s.t. it is still the median-voter. The change of preferences of the median voter in the direction of public goods (β increases; α decreases) directly leads to a shift in the proposed combinations by parties and a higher provision of public goods. People are willing to abstain from private goods in order to get a higher level of public goods. An increasing budget results in the same shift. In this case, the median voter is not willing to abstain from private goods but uses some part of the additional income to demand more public goods. As simple as this conclusion might look, it is essential. If income increases

– which is the case for most countries in the world and the world average[5] – and the median income is affected significantly, the demanded amount of public goods increases automatically. In a competitive political system with two parties, this shift results in an increased provision of public goods. Using this calculus, observed changes of the provision of public goods in reality can be easily explained by changes of the willingness to pay caused by an increasing income.

Let us apply this result on the environment as an example for a public good: In a situation with a high level of environmental protection but low income people are willing to accept some destruction of the environment to get more private goods. The level of the environment is "too high", and individuals would vote for the party that is providing a lower level of public goods. This would imply that the individuals can consume more private goods. Thus, it is only logical that higher destruction of the environment can be observed in low income countries. However, with an increasing income, we expect a turning point where the level of protection of the environment will increase automatically: The demanded level of the public good by the median voter is now higher than the actual level of the public good. From this turning point on, the destruction of the public good decreases, provided that income is further increasing. This result is also known as environmental Kuznets curve and empirically controversial (Schneider 2022; Chu 2021; Moomaw and Unruh 1997). Deviations from the constructed model could be caused by different reasons, e.g.:

1. The illustrated model holds only for two parties. It is unclear, how different political systems involving more parties could change the results.

2. Not only income but also preferences might shift over time as they may be affected by exogenous shocks. Furthermore, there is not just one public good, s.t. also preferences between different public goods might change.

3. Depending on the size of the public goods, the free-rider problem remains and the willingness to pay is not revealed. For larger public goods like climate, there is no

[5] World Development Indicators of the Worldbank show that GDP per capita, PPP (constant 2017 international $) comparing years 2000 and 2020 increased for 151 out of 182 countries, for which these data are available. For this period world average increased by 46%.

entity that can force individuals to pay for this good. Thus, a higher income is not automatically leading to a higher provision of the public good.[6]

Beside the discussion on a specific public good, literature shows a positive correlation between income and the provision of the public goods (Inman 1978; Deacon 2009).

So far, we have focused on the median voter. As long as this median voter remains constant, a change in the preferences or budget of other voters within the society does not affect the provision of public goods. Only if preferences or income of the median voter change, the provision of public goods increases or decreases. If the income of the 40 percent of the population that has a higher willingness to pay than the median voter increases, the provision of public goods is not changing. Suppose the income of the 40 percent of the population that has a lower willingness to pay than the median voter decreases. In this case, the provision of public goods is not changing as long as they can still afford the provided combination. This example gives an intuition why redistribution of income can be Pareto-superior to the status quo. Transferring income from the rich to the poor can lead to a higher provision of public goods if the median voter is affected. A higher utility for people having a higher willingness to pay for the public good than the median voter might arise. Therefore, the lost utility due to lower consumption of private goods must be smaller than the gained utility due to the increased public good provision. Figure 12 shows that a budget reduction (with budget being shifted to the median voter) can lead to a higher utility due to the provision of more public goods (combination at point Q gives higher utility than combination P at lower budget).

[6]However, as the willingness to pay for the public good is rising, it is likely that people support parties willing to solve the free-rider problem on a higher political level in cooperation with other countries. This might also result in a higher provision of larger public goods.

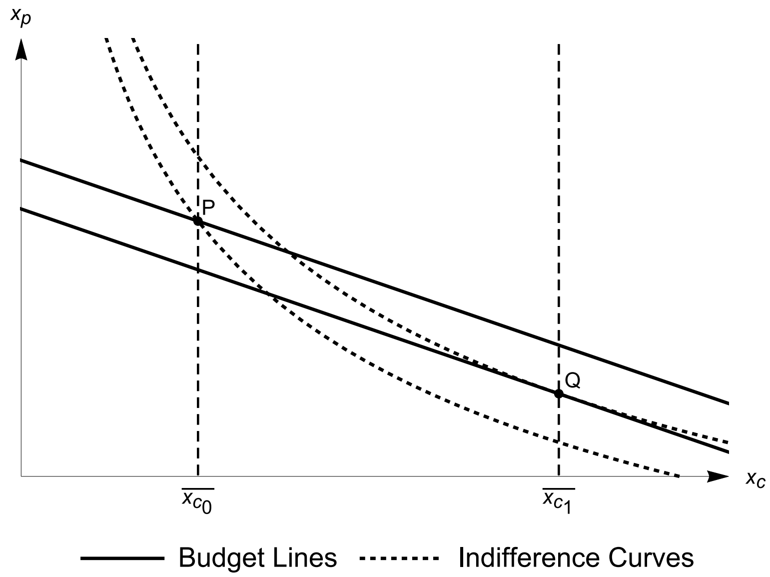

Figure 12: Pareto-efficient redistribution

Such a transfer could also be achieved by changing the tax from a lump sum tax to a proportional or even progressive tax on income. For the model, this would imply different prices for the same amount of the public good depending on a person's income.

4 Conclusion and Extensions

In contrast to household optimization for private goods, individuals cannot choose the amount of the public good they want to consume. The government determines it. The only way to change the provided combination of public goods is through voting. Parties propose different quantity-price combinations of public goods. Indifference curves can be implemented in the price-quantity diagram to compare these combinations and investigate what party an individual would vote for. These indifference curves show different utility levels of the individual and can be used to compare every combination of price and quantity of a public good (which implies that the remaining budget is spent on private consumption).

In a competitive political market, a party proposing a combination at a price higher than the production cost of the public good will always lose the election. Another party can offer the same amount at a lower price. Since this combination gives a higher utility, a party offering a combination with “inefficiencies” will always be sorted out by the political competition. In a two party system, where voters have different budgets, parties have an incentive to change their proposed combination until it represents the utility-maximizing combination of the median voter. Only by providing this combination, they can win the election. This is true even for imperfect competition.

These results imply that a change in the provision of public goods can only be achieved by changing the median voter’s preferences or budget. For growing economies (and a growth which is at least as uniformly distributed such that it affects the median voter) with a competitive political system, this means that more public goods are demanded and provided automatically. No change of preferences is needed to increase the level of the public good. Furthermore, redistribution or the unequal burden of tax for the provision of public goods can lead to a Pareto-superior outcome: If the utility decrease caused by redistribution is lower than the gain due to a higher provision of public goods, everybody’s utility increases compared to the status quo.

Having explained the potential implications of the willingness to pay based median-voter model it is important to focus on possible extensions. As explained, the voters’ willingness to pay is the only channel that can change the provision of public goods. In reality, we can observe that individuals donate money to interest groups to change the provision of public goods. In the given model this should not affect the provision of public goods as the budget and preferences of the median voter are not changing. This result remains valid as long as perfect information is assumed. In reality, this assumption is not fulfilled though. The lack of knowledge is twofold then, affecting the voters and the parties.

First, let us focus on interdependencies between public and private goods. For example, certain private goods could be provided for a lower price if public goods have a high level (e.g., a skiing holiday is cheaper if there is less global warming because the slopes do not have to be snowed artificially). The utility of other private goods might depend on the level of the public goods (e.g., clean air and a holiday trip). In this case, the willingness to pay for public goods is influenced by its effect on private goods. The higher the effect of public goods on private goods, the higher the willingness to pay for public goods. Thus, information on this relationship plays a crucial role in the provision of public goods by parties. Interest groups could use their budget to influence voters, hence their willingness to pay by showing that the effect of the level of a public good on private goods is lower or higher than actually expected. The given information might change the utility calculus of individuals and affect the willingness to pay of the median voter. Thus, this leads to a higher or lower provision of a public good. As a consequence, it can be reasonable to abstain from spending certain parts of the budget on private goods and invest it into interest groups instead. If the provided information changes the provision of public goods, this might increase utility to a higher level as the less consumed private goods decrease it.

Second, missing information leads to the problem that parties do not know "who" the median voter is. This uncertainty can be exploited by interest groups. They can contact politicians directly to convince them that the median voter wants more or less of a specific public good. Furthermore, they can use media, such as television, newspaper or social media, to make it appear that a particular measure is or is not in line with the median voter. This can also change the perception of politicians (and parties) what and which amount of public goods the median voter wants to have and hence affect its provision. Thus, with a lack of information parties will provide the combination of the "perceived" median voter.

The introduced extension can close the gap between the median-voter theorem and theories on interest groups (Olson 1971; Becker 1983) and gives an important approach for further research. Furthermore, this model provides the opportunity to link two theories of determining the demand for public goods (Batina and Ihori 2005). Beside the median-voter theorem another way of determining the demand for public goods is gathering information about the willingness to pay of individuals in an incentive compatible framework (Clarke 1971; Groves 1973). As both approaches can deal with the willingness to pay for public goods of individuals it is worth to explore whether they lead to the same results.

References

Adams, James, Samuel Merrill, and Roi Zur. 2020. "The Spatial Voting Model." In *The SAGE Handbook of Research Methods in Political Science and International Relations,* edited by Luigi Curini and Robert Franzese, 205–223. London: SAGE Publications.

Arrow, Kenneth J., and Gérard Debreu. 1954. "Existence of an equilibrium for a competitive economy." *Econometrica* 22 (3): 265–290.

Axelrod, Robert M. 1984. *The evolution of cooperation.* New York: Basic Books.

Batina, Raymond G., and Toshihiro Ihori. 2005. *Public Goods: Theories and Evidence.* Berlin and Heidelberg: Springer.

Becker, Gary S. 1968. "Crime and punishment: An economic approach." *Journal of Political Economy* 76 (2): 169–217.

———. 1983. "A theory of competition among pressure groups for political influence." *The quarterly journal of economics* 98 (3): 371–400.

Black, Duncan. 1948. "On the Rationale of Group Decision-making." *Journal of Political Economy* 56 (1): 23–34.

———. 1958. *The theory of committees and elections.* Cambridge: University Press.

Bornstein, Gary, Ido Erev, and Orna Rosen. 1990. "Intergroup competition as a structural solution to social dilemmas." *Social Behaviour* 5 (4): 247–260.

Brubaker, Earl R. 1975. "Free Ride, Free Revelation, or Golden Rule?" *The Journal of Law and Economics* 18 (1): 147–161.

Chu, Lan Khanh. 2021. "Economic structure and environmental Kuznets curve hypothesis: new evidence from economic complexity." *Applied Economics Letters* 28 (7): 612–616.

Clarke, Edward H. 1971. "Multipart pricing of public goods." *Public Choice* 11 (1): 17–33.

Deacon, Robert T. 2009. "Public good provision under dictatorship and democracy." *Public Choice.*

Downs, Anthony. 1957. *An economic theory of democracy.* New York: Harpon and Row.

Gogoll, Niklas, and Felix Schlieszus. 2021. *Why Being a Frequent Flyer and an Environmental Activist is no Contradiction: The Willingness to Pay for Public Goods and its Components.* University of Bayreuth, Bayreuth. Accessed August 7, 2022. https://www.vwl5.uni-bayreuth.de/pool/dokumente/Working-Paper/Gogoll-_-Schlieszus---Frequent-Flyer-Environmental-Activist.pdf.

Groves, Theodore. 1973. "Incentives in teams." *Econometrica* 41 (4): 617–631.

Guagnano, Gregory A., Thomas Dietz, and Paul C. Stern. 1994. "Willingness to Pay for Public Goods: A Test of the Contribution Model." *Psychological Science* 5 (6): 411–415.

Holcombe, Randall G. 1989. "The median voter model in public choice theory." *Public Choice* 61 (2): 115–125.

Hotelling, Harold. 1929. "Stability in competition." *The economic journal* 39 (153): 41–57.

Inman, Robert P. 1978. "Testing political economy's 'as if' proposition: is the median income voter really decisive?" *Public Choice* 33 (4): 45–65.

Kreps, David M., and Robert Wilson. 1982. "Reputation and imperfect information." *Journal of Economic Theory* 27 (2): 253–279.

Mani, Ankur, Iyad Rahwan, and Alex Pentland. 2013. "Inducing peer pressure to promote cooperation." *Scientific reports* 3 (1735).

Margolis, Howard. 1983. "Selfishness, Altruism, and Rationality: A Theory of Social Choice." *American Political Science Review* 77 (4): 1130–1131.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green. 1995. *Microeconomic theory.* New York: Oxford University Press.

Moomaw, William R., and Gregory C. Unruh. 1997. “Are environmental Kuznets curves misleading us? The case of CO 2 emissions: MOOMAW, WILLIAM R.; UNRUH, GREGORY C.” *Environment and Development Economics* 2 (4): 451–463.

Niskanen, William A. 1971. *Bureaucracy and representative government.* Chicago, Ill.: Aldine-Atherton.

Nowak, Martin A., and Karl Sigmund. 2005. “Evolution of indirect reciprocity.” *Nature* 437 (7063): 1291–1298.

Olson, Mancur. 1971. *The logic of collective action: Public goods and the theory of groups.* 2. ed. Cambridge, MA: Harvard University Press.

Ostrom, Elinor. 1990. *Governing the commons: The evolution of institutions for collective action.* Cambridge: Cambridge University Press.

Rowley, K., Charles. 1984. “The Relevance of the Median Voter Theorem.” *Journal of Institutional and Theoretical Economics* 140 (1): 104–126.

Samuelson, Paul A. 1954. “The Pure Theory of Public Expenditure.” *The Review of Economics and Statistics* 36 (4): 387–389.

Sandler, Todd. 1992. *Collective action: Theory and applications.* Ann Arbor, MI: University of Michigan Press.

Schneider, Nicolas. 2022. “Sustainable development or environmental Kuznets curve: some unresolved controversies.” *International Journal of Environmental Studies,* 1–4.

Taylor, Michael. 1987. *The possibility of cooperation.* Studies in rationality and social change. Cambridge Cambridgeshire and New York: Cambridge University Press.

Ule, Aljaz, Arthur Schram, Arno Riedl, and Timothy N. Cason. 2009. “Indirect punishment and generosity toward strangers.” *Science* 326 (5960): 1701–1704.

A Appendix

A.1 Utility & Demand Curve

Lemma A.1. *Combinations on the inverse demand curve with lower prices (higher amount of x_c) gain a higher utility than combinations on the inverse demand curve at higher prices (lower amount of x_C).*

$$U(D_{x_c}(p_{c1})) > U(D_{x_c}(p_{c2}))\ \textit{if}\ p_{c1} < p_{c2}$$

Proof. Using the utility function depending only on the collective good x_c and including the budget restriction (equation 5) yields

$$U(x_c) = \left(\frac{m - x_c p_c}{p_p}\right)^{\alpha} x_c^{\beta}. \tag{5}$$

Replacing the price of the collective good with the inverse demand function

$$D_{p_c}^{-1}(x_c) = \frac{\beta m}{(\alpha + \beta) x_c},$$

s.t. only the utility of the points on the inverse demand function is observed, yields

$$U(x_c) = \left(\frac{m - \frac{\beta m}{\alpha+\beta}}{p_p}\right)^{\alpha} {x_c}^{\beta}.$$

Differentiating this function with respect to x_c gives

$$\frac{dU(x_c)}{dx_c} = \left(\frac{m - \frac{\beta m}{\alpha+\beta}}{p_p}\right)^{\alpha} \beta {x_c}^{\beta-1}.$$

Using assumptions AS1 and AS2 it can be concluded that the derivative is positive and if x_c is increasing, which implies a price decrease in p_c, the utility is increasing. □

A.2 Lower Price at Constant Amount

Lemma A.2. *For a given amount of the collective good, the utility is higher with a lower price for* p_c*:*

$$U(p_{c_1}) > U(p_{c_2}) \text{ if } p_{c_1} < p_{c_2}$$

Proof. Using the utility function depending only on the collective good x_c and including the budget restriction (equation 5) yields

$$U(x_c) = \left(\frac{m - x_c p_c}{p_p}\right)^{\alpha} x_c^{\beta}. \tag{5}$$

Differentiating this function with respect to p_c leads to

$$\frac{dU(x_c)}{dp_c} = -\frac{\alpha {x_c}^{\beta+1} \left(\frac{m-p_c x_c}{p_p}\right)^{\alpha-1}}{p_p}.$$

This term is negative as all variables are positive and $m - p_c x_c \geq 0$ must hold s.t. budget restriction holds. It can be concluded that if the price of the collective good p_c is increasing for a fixed amount of the collective good x_c, the utility is decreasing. □

A.3 Utility beside Demand Function at Constant Price

Lemma A.3. *For any given price of the collective good p_c, the utility decreases for a higher or lower amount of the collective good x_c beside the combination on the demand function x_c^* monotonously:*

$$U(x_c^* + a) > U(x_c^* + b) \text{ if } a < b \text{ and } a > 0 \ \& \ b > 0$$

and

$$U(x_c^* - a) > U(x_c^* - b) \text{ if } a < b \text{ and } a > 0 \ \& \ b > 0$$

Proof. Using the utility function only depending on the collective good x_c and including the budget restriction (equation 5) yields

$$U(x_c) = \left(\frac{m - x_c p_c}{p_p}\right)^\alpha x_c^\beta. \tag{5}$$

Substituting x_c with the demand function (equation 4), which is providing the utility maximizing amount of the collective good x_c^* for a given price p_c, and adding x_c leads to

$$U(x_c^* + x_c) = \left(\frac{\beta m}{p_c(\alpha+\beta)} + x_c\right)^\beta \left(\frac{m - p_c\left(\frac{\beta m}{p_c(\alpha+\beta)} + x_c\right)}{p_p}\right)^\alpha.$$

Differentiating this function with respect to x_c yields

$$\frac{dU(x_c^* + x_c)}{dx_c} = \frac{{p_c}^2 x_c(\alpha+\beta)^3 \left(\frac{\beta m}{p_c(\alpha+\beta)} + x_c\right)^\beta \left(\frac{\frac{\alpha m}{\alpha+\beta} - p_c x_c}{p_p}\right)^\alpha}{(p_c x_c(\alpha+\beta) - \alpha m)(p_c x_c(\alpha+\beta) + \beta m)}. \tag{8}$$

For $x_c > 0$ this derivative (equation 8) is negative. All variables are positive s.t. in the numerator only the part

$$\frac{\alpha m}{\alpha+\beta} - p_c x_c$$

can be negative. Using assumption AS2 this term simplifies to

$$\alpha m - p_c x_c. \tag{9}$$

For the denominator the only term which can be negative is

$$p_c x_c(\alpha + \beta) - \alpha m.$$

Using assumption AS2 this term simplifies to

$$p_c x_c - \alpha m. \tag{10}$$

If equation 9 is positive, equation 10 is negative or vice versa.[7] Thus, the whole term is negative, which implies that utility decreases for values of x_c above 0. It can be concluded, that for any amount higher than the amount on the demand curve, the utility for an individual is lower at any given price p_c.

For $x_c < 0$ the derivative (equation 8) is positive. In addition to the calculus above, in the numerator x_c is negative, such that the whole term turns positive. Furthermore, in the numerator term

$$\frac{\beta m}{p_c(\alpha + \beta)} + x_c$$

can be negative. Using assumption AS2 this term simplifies to

$$\frac{\beta m}{p_c} + x_c. \tag{11}$$

In the denominator term

$$p_c x_c(\alpha + \beta) + \beta m$$

[7]If $p_c x_c = \alpha m$ the utility is zero for any x_c and no utility maximum exists.

can be negative. Using assumption AS2 this term simplifies to

$$p_c x_c + \beta m. \tag{12}$$

As equation 11 and 12 have the same sign, the whole term remains positive.[8] This implies that utility decreases for values of x_c below 0. It can be concluded, that for any amount lower than the amount on the demand curve, the utility for an individual is lower at any given price p_c. □

[8] For the case $p_c x_c = \beta m$ no derivative exists within the range of real numbers. However, this case can be ignored as this condition would imply (having negative value of x_c) that at least one of the variables p_c, β or m is negative, which is not allowed.

A.4 Monotonicity of Utility on Increasing Marginal Cost Curve

Lemma A.4. *For any combination beside the utility maximizing combination on the marginal cost curve, the utility decreases monotonously on the increasing marginal cost curve:*

$$U(A_{x_c}(x_c^* + a)) > U(A_{x_c}(x_c^* + b)) \text{ if } a < b \text{ and } a > 0 \text{ \& } b > 0$$

and

$$U(A_{x_c}(x_c^* - a)) > U(A_{x_c}(x_c^* - b)) \text{ if } a < b \text{ and } a > 0 \text{ \& } b > 0$$

Proof. At first, it is necessary to calculate the utility maximum on marginal cost curve: The utility maximum must be the tangential point with the lowest indifference curve (respective highest utility). Therefore, two conditions must be fullfilled:

1. The point of the marginal cost function and indifference curve must be identical. The marginal cost function is given by

$$A_{x_c}^{-1}(x_c) = x_c. \tag{13}$$

The indifference curve in price-quantity diagram is (see equation 6)

$$I_{\overline{U}}(x_c) = \frac{m - p_p(\frac{\overline{U}}{x_c^\beta})^{\frac{1}{\alpha}}}{x_c}. \tag{6}$$

Equating equation 13 and 6 yields

$$x_c = \frac{m - p_p(\frac{\overline{U}}{x_c^\beta})^{\frac{1}{\alpha}}}{x_c}.$$

Solving for the constant utility value yields

$$\overline{U} = \left(\frac{x_c \left(\frac{m}{x_c} - x_c \right)}{p_p} \right)^{\alpha} {x_c}^{\beta}. \tag{14}$$

2. The slope of of the marginal cost function and the slope of the indifference curve must be identical. The slope of the marginal cost function is

$$\frac{dA_{x_c}^{-1}(x_c)}{dx_c} = 1. \tag{15}$$

The slope of the indifference curve is

$$\frac{dI_{\overline{U}}(x_c)}{dx_c} = \frac{p_p(\alpha + \beta) \left(\frac{\overline{U}}{{x_c}^{\beta}} \right)^{\frac{1}{\alpha}} - \alpha m}{\alpha {x_c}^2}. \tag{16}$$

Equating equations 15 and 16 yields

$$1 = \frac{p_p(\alpha + \beta) \left(\frac{\overline{U}}{{x_c}^{\beta}} \right)^{\frac{1}{\alpha}} - \alpha m}{\alpha {x_c}^2}.$$

Solving for the utility value yields

$$\overline{U} = \left(\frac{\alpha \left(m + {x_c}^2 \right)}{p_p(\alpha + \beta)} \right)^{\alpha} {x_c}^{\beta}. \tag{17}$$

Equating 14 and 17 and solving for x_c yields the maximum utility on the marginal cost curve

$$x_c^* = \frac{\sqrt{\beta}\sqrt{m}}{\sqrt{2\alpha + \beta}}. \tag{18}$$

Using the utility function only depending on the collective good x_c and including the budget restriction (equation 5) yields

$$U(x_c) = \left(\frac{m - x_c p_c}{p_p}\right)^{\alpha} x_c^{\beta}. \tag{5}$$

Substituting the price of the collective good p_c with the marginal cost function leads to

$$U(x_c) = \left(\frac{m - x_c x_c}{p_p}\right)^{\alpha} x_c^{\beta}. \tag{5}$$

The second derivative of this function is

$$\frac{d^2U(x_c)}{dx_c^2} = \frac{(-2m{x_c}^2(2\alpha\beta + \alpha + (\beta - 1)\beta) + {x_c}^4(2\alpha + \beta - 1)(2\alpha + \beta) + (\beta - 1)\beta m^2)}{(m - {x_c}^2)^2} * \frac{{x_c}^{\beta-2}\left(\frac{m-{x_c}^2}{p_p}\right)^{\alpha}}{(m - {x_c}^2)^2}.$$

Substituting x_c with the utility maximum x_c^* leads to

$$\frac{d^2U(x_c^*)}{dx_c^2} = -\frac{2^{\alpha}(2\alpha + \beta)^2 \left(\frac{\sqrt{\beta}\sqrt{m}}{\sqrt{2\alpha+\beta}}\right)^{\beta} \left(\frac{\alpha m}{2\alpha p_p + \beta p_p}\right)^{\alpha}}{\alpha m}.$$

As all variables are positive this term is negative. This implies that the optimum is a global maximum and the utility decreases monotonously beside its maximum on the marginal cost curve.

□

A.5 Function of Inefficiency Maximizing Combinations

Lemma A.5. *Given a constant marginal cost of production function, a utility function and budget restriction,*

$$p_c = \frac{\beta m - \alpha c_c x_c}{\beta x_c} \tag{19}$$

is giving the curve of inefficiency maximizing points.

Proof. The indifference curves in the price-quantity diagram (equation 6) are given by

$$I_{\overline{U}}(x_c) = \frac{m - p_p(\frac{\overline{U}}{x_c^\beta})^{\frac{1}{\alpha}}}{x_c}. \tag{6}$$

The marginal cost function is given by

$$A_{x_c}^{-1}(x_c) = c_c,$$

where c_c is a positive real number. Any point between the indifference curve for the respective utility value and the supply function would lead to a higher utility level of the mentioned individual and thus in winning the election. The indifference curve itself would lead to a draw. In this area the combination having the highest total inefficiencies can be calculated. This can be achieved by multiplying the price given by the indifference curve minus the cost per unit, which is given by the marginal cost function, times the proposed amount of the collective good

$$\Pi(x_c) = \left(\frac{m - p_p(\frac{\overline{U}}{x_c^\beta})^{\frac{1}{\alpha}}}{x_c} - c_c \right) * x_c.$$

In order to get the maximum inefficiencies this term is differentiated with respect to x_c and set equal to zero

$$\frac{d\Pi(x_c)}{dx_c} = \frac{\beta p_p \left(\overline{U} {x_c}^{-\beta}\right)^{1/\alpha}}{\alpha x_c} - c_c = 0. \tag{20}$$

Using the utility function depending only on the collective good x_c and including the budget restriction (equation 5)

$$U(x_c) = {x_c}^{\beta} \left(\frac{m - p_c x_c}{p_p} \right)^{\alpha} \tag{5}$$

to substitute the constant utility-value in equation 20, simplifying the term, solving it to p_c yields

$$p_c = \frac{\beta m - \alpha c_c x_c}{\beta x_c}. \tag{21}$$

This term provides for every amount of the collective good x_c the price p_c which maximizes inefficiencies for a party for a respective utility value. □

A.6 Utility Maximization on the path of inefficiency maximizing combinations for all voters

Lemma A.6. *Every combination on the inefficiency maximizing path which leads to a higher utility of the median voter, also leads to a higher utility of the other voters:*

$$U_2(\Pi_{max}(x_{c_1})) > U_2(\Pi_{max}(x_{c_2})) \text{ } \textit{if} \text{ } U_{MV}(\Pi_{max}(x_{c_1})) > U_{MV}(\Pi_{max}(x_{c_2}))$$

Proof. Let the budget of a second individual be m_2. Preferences remain the same, s.t. the utility function U_2 only changes as private goods x_p are replaced by the remaining budget not used for collective goods.

$$U_2 = x_p^{\alpha} x_c^{\beta} = x_c{}^{\beta} \left(\frac{m_2 - p_c x_c}{p_p} \right)^{\alpha}$$

results as utility function. For p_c the inefficiency maximizing function (equation 19) is substituted

$$U_2 = x_c{}^{\beta} \left(\frac{m_2 - \frac{\beta m - \alpha c x_c}{\beta}}{p_p} \right)^{\alpha_2}.$$

Differentiating with respect to x_c yields

$$\frac{dU_2(x_c)}{dx_c} = \frac{\alpha^2 k x_c{}^{\beta} \left(\frac{m_2 - \frac{\beta m - \alpha c x_c}{\beta}}{p_p} \right)^{\alpha - 1}}{\beta p_p} + \beta x_c{}^{\beta - 1} \left(\frac{m_2 - \frac{\beta m - \alpha c x_c}{\beta}}{p_p} \right)^{\alpha}.$$

Using assumtions AS1 and AS2 the derived value must be positive for an increasing x_c and a solution within the range of real numbers. U_2 is increasing with x_c, which implies that voters will always choose the point on the path where corresponding combination is containing more x_c, hence also the median voter has the higher utility. □